any matrix f can be expressed as a function of the variables α, β instead of p, q, just as in classical theory we can use (6) as well as (7). The correspondence principle then follows from the asymptotic agreement of classical and quantum canonical transformations. It seems, nevertheless, of interest to have proved the correspondence principle as directly as possible without appeal to iterated transformations.

[1] P. A. M. Dirac, *Proc. Roy. Soc. London*, **113A**, 621 (1927).

[2] P. Jordan, *Zeits. Physik*, **40**, 809 and **44**, 1 (1927).

[3] G. Wentzel, *Ibid.*, **38**, 518 (1926).

[4] L. Brillouin, *J. Physique*, **7**, 353 (1926).

[5] C. Eckart, *Proc. Nat. Acad. Sci.*, **12**, 684 (1926).

[6] Born, Heisenberg and Jordan, *Zeits. Physik*, **35**, 563 (1926).

[7] Cf., for instance, Muir, *Theory of Determinants*, vol. II, p. 230. The identity is due originally to Jacobi, *Crelle's Journ.*, **27**, 199 (1846), or *Gesammelte Werke*, iv, 317.

[8] H. A. Kramers, *Zeits. Physik*, **39**, 828 (1926).

[9] P. Jordan, *Ibid.*, **38**, 513 (1926); and **41**, 797 (1927).

EXPERIMENTS WITH MODIFIED MUCRONATE ELECTRODES

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Communicated January 12, 1928

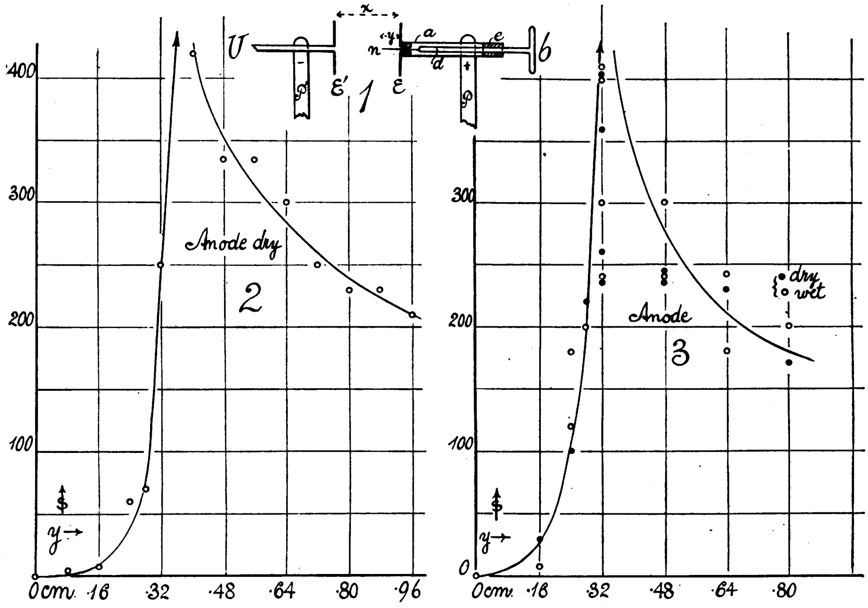
1. *Correction.*—In an earlier paper* I found the anode behaving in a way indistinguishable from the cathode. Many repetitions of the experiment since have shown that this is not the case. What probably happened was a spontaneous change of the polarity of the electrical machine for which I was unprepared. Hence the cathode behavior was inadvertently measured twice. The correct graphs are given in the following paragraphs.

2. *Apparatus.*—This is essentially the same as before, consisting of the electrodes E E' (about 2 cm. in diameter) of the spark gap x of a small electrostatic machine. The quill tube from E' leads to the interferometer U-tube beyond U, for measuring the pressure of the electric wind (s, roughly in 10^{-6} atm.). The electrode E is provided with a micrometer screw db carrying the needle n, whose extrusion y beyond the electrode is thus measurable. P and P' are insulated posts and c a constriction of the pipe a, carrying the electrode E and the nut e of the screw. A similar arrangement for the cathode is also provided (not shown).

3. *Moist and Dry Electrodes.*—Observations made in the dark with electrodes moistened, for instance, with glycerine, recorded very marked differences of behavior. There is not room to describe these here further

than to state that the diffuse brilliant spark succession changed, in general, to a dull gray arc which retained, however, many of the properties of the sparks. Thus when the needle was critically set (electric wind just about to change into a diffuse spark succession, y-value of the cusps in the sy graphs) the spark gap may subserve the purpose of an electroscope to a negatively charged rod, but is indifferent to a positively charged rod;** etc.

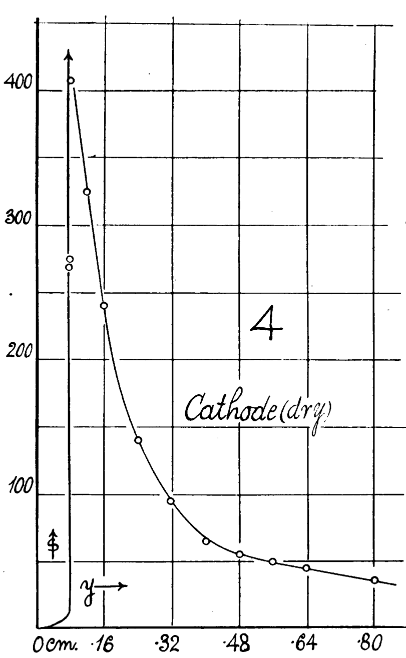
4. *The sy Graphs.*—It was supposed, therefore, that the wind pressure phenomena would show an equally striking behavior; but this is not the case as the graphs, figures 2 to 5, indicate.

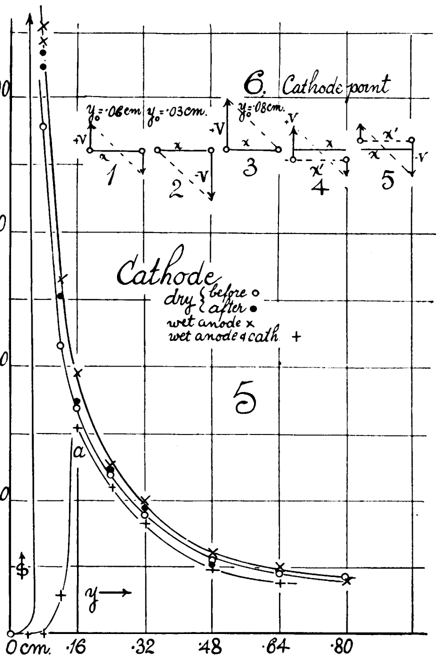


The electrodes were carefully overhauled and readjusted in parallel. figures 2 and 3 give the measurements for the anodal mucronate electrode. High cusps (quite as high as in the cathodal cases) are in evidence. The cusps for the positive electrons lie at the high extrusion value $y = 0.32$ cm. (about) characterizing the anodal convection current. Figure 3 shows that no valid distinction can be made between wet and dry electrodes, the straggling displacements being due to the violent oscillations of fringe displacements in all cases. At the anode, therefore, convection and spark discharges irregularly alternate, pressures s dropping from high values to zero in rapid succession. Hence the restlessness of the fringes, making it difficult to find the true displacement, s.

For the corresponding cathode graphs this oscillation of fringes is far less turbulent, even if also present. The negative electrons thus issue more steadily. The curve for dry electrodes, figure 4, with its low and characteristic $y = 0.06$ cm. (about) has the usual high cusp, $s = 400$, in this respect, however, not exceeding the anode. In figure 5 the cases of wet and dry electrodes are contrasted. The difference is too small to be of any discriminating value, and wet and dry electrodes again behave alike. It is true that the graph for both electrodes wet departs from the others (cusp here at $y = 0.16$ cm., and low); but this is due (as I found by trial) to the irregularity of the liquid surface (drops forming) and not to non-incidental causes.

Thus we have a similarity of behavior for anode and cathode mucronate electrode, wet or dry, throughout, with the difference that $y > 0.32$ cm. for the anode, while $y < 0.08$ cm. for the cathode mucronate extrusion.





5. *Needles of Different Metal.*—A copper and a zinc needle were compared with the steel needle, but no difference in behavior was appreciable. The change from convection current to spark discharge (cusp) occurred at the cathode in all cases at about $y = 0.06$ cm., an uncertainty of set of about 0.02 cm. (sparks striking earlier or later incidentally) being inevitable. The height of cusps was also about the same, and this happened here to reach the exceptionally high pressure values exceeding 0.04 cm. of mercury.

6. *Summary.*—To get some idea of the questions here involved we may assume that the potential difference of the 2 cm. spark gap x is 50 kv.

and that the axial field is uniformly of the value 25 *kv.*/cm.; furthermore, that when the needle point passes outward from the electrodes, it encounters potential differences, proportional to the extrusion y. The surface density at the needle point and the space charges unfortunately cannot be given. Hence in case of the cathode point, where the critical set is $y_0 = 0.06$ cm., the potential difference $\Delta V = 25 \times 0.06 = 1.5$ *kv.* is needed to secure a steady emission of negative electrons, resulting in a uniform negatively ionized convection current with a wind pressure (at $x = 2$ cm.), which may reach 0.4 mm. of mercury. In case of the anode, however, the corresponding critical extrusion is $y_0 = 0.24$ cm. and the potential difference needed for the steady emission of positive electrons, therefore $\Delta V = 25 \times 0.24 = 6$ *kv.* The maximum pressure of the positively ionized wind is about the same as for the cathode.

Apart from the reduction of field to be associated with the steady convection current, this point of view suggests no adequate reason for the very rapid decrease of wind pressure (s) from a cusp when the extrusion y further increases; but if we admit it tentatively, figure 6 may be used to record the more important observations. Diagram 1 shows the potential at the electrodes (originally equal and opposite), the field and the constant spark distance x while the critical y_0 is annotated. In diagram 2 the positive electrode is earthed and the maximum negative potential active; y_0 has now dropped to 0.03 cm. In diagram 3, in which the cathode is earthed, y_0 has risen to 0.08 cm. Supposing the needle critically set (y_0), the result in all cases of y decrement is an immediate change of the convection current of negative ions, with high s, into a diffuse spark discharge with $s = 0$. In conformity with the high positive potential of case 3, it seems reasonable to assume that positive ions are now discharged even if the field is not necessarily changed. In diagram 4 I have attempted to show the effect when $y_0 = 0.06$ cm. of a negatively charged body near the spark gap superimposing a negative potential on the potentials of the field. The result is a lower fiducial line x' in place of x, so that the positive potential is again virtually increased. The negative convection current passes into a positive spark succession as in case No. 3. Finally diagram 5 suggests the corresponding rise of the fiducial line to x', owing to the presence of a positively charged body near the spark gap. The effect of this is inappreciable as we here encounter relations similar to those of No. 2 with high negative potential, where the convection current strikes when $y_0 = 0.03$. It is noteworthy that $y_0 = 0.08, 0.06, 0.03$ cm., respectively, suffice for initiating the steady convection of negative ions, while the negative potential increases from zero to the largest negative value for the same nominal field.

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* These PROCEEDINGS, **13,** 1927, pp. 503–5.

** *Science,* **66,** 1927, pp. 658–9.